Instructions

1. The use of calculators, books, or notes is not allowed.

2. Provide clear arguments and computations for all your answers: only answering "yes", "no", or "42" is not sufficient.

3. The total score for all questions equals 45 points. If p is the number of marks then the exam grade is $G = 1 + p/5$.

Problem 1 (15 points)

For a fixed number $\theta \in \mathbb{R}$ consider the following function:

$$f(z) = \operatorname{Re}(z^2) + e^{i\theta} \operatorname{Im}(z^2).$$

Use the Cauchy-Riemann equations to determine all values of $\theta \in \mathbb{R}$ such that f is differentiable at every point $z \in \mathbb{C}$.

Problem 2 (15 points)

Determine a real-valued function $\phi(x, y)$ which is harmonic in the domain

$$D = \{(x, y) \in \mathbb{R}^2 : x > 0\}$$

and satisfies the following condition:

$$\phi(0, y) = -\frac{y}{1 + y^2}.$$

Hint: consider the complex function $f(z) = 1/(z + 1)$.

Problem 3 (15 points)

Compute the partial fraction decomposition for the following rational function:

$$R(z) = \frac{2z^2 - 6z - 11}{(z^2 + z - 6)(z - 2)}.$$

End of test (45 points)